Addition

Please note: some images show 'ones' as units. It is important for consistency that we all call them 'ones'.
We now follow a mastery curriculum and ensure the children are taken through the curriculum in small steps. We follow White Rose Hub planning and resources and complement it with other necessary resources and activities.
We try to use Mastery principles that are outlined in NCTEM Calculation Guidance for Schools –
Conceptual variation is key to the child's learning and understanding.
https://www.ncetm.org.uk/public/files/25120980/NCETM+Calculation+Guidance+October+2015.pdf

Vocabulary -

Place Value, sum, total, parts and wholes, plus, addition, add, +, more, plus, make, 'is equal to' 'is the same as', equals = same as, put together, more than, total, altogether, distance between, most, pattern, odd, even, digit, forward counting on, sign, thousands, hundreds, tens, ones, partition, double, number line, column, boundary, vertical, carry, expanded, compact, inverse.
Near multiple of 10, one more, two more… ten more… one hundred more, one thousand more, decimal place, decimal point, tenths, hundredths, thousandths

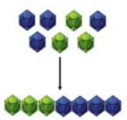
Key Questions

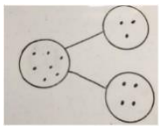
How many more to make…? How many more is… than…? How much more is…? How many altogether?
I add …more. What is the total? How much more is…? One more, two more, ten more…
What can you see here? Is this true or false? What is the same? What is different? What do you notice? What patterns can you see?
If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 – 17 = 2; 19 – 2 = 17; 190 – 20 = 170 etc).
When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line? Can you convince me? How do you know?

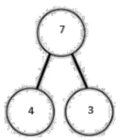
EYFS / Year 1

Children will need opportunities to look at and talk about different models and images as they move between representations.

Combining two parts to make a whole: part whole model.
Concrete; Pictorial; Abstract;







Year 2

Continue to use Concrete, Pictorial and Abstract strategies to all areas. Children will need opportunities to look at and talk about different models and images as they move between representations.

Adding three single digits, children should practise addition to 20, 50 and 100 to become increasingly fluent. They should use the facts they know to derive others, e.g using 7 + 3 = 10 to find 17 + 3= 20, 70 + 30 = 100

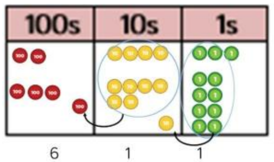
As well as number lines, 100 squares are used to explore patterns in calculations such as 74 +11, 77 + 9 encouraging children to think about

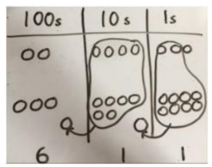
Year 3

Continue to use Concrete, Pictorial and Abstract strategies to all areas. Children will need opportunities to look at and talk about different models and images as they move between representations.

Column method regrouping. Using place value counters (up to 3 digits)

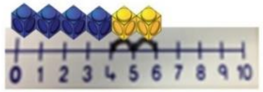
Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



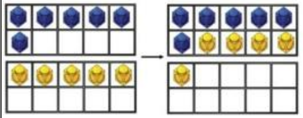


Mental methods

Starting at the bigger number and counting on- using cubes.



Regrouping to make 10 using ten frame.



<u>+ = signs and missing numbers</u>

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

2 = 1+ 1

2 + 3 = 4 + 1

Missing numbers need to be placed in all possible places.

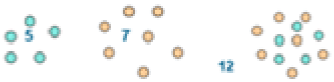
3 + 4 = □   □ = 3 + 4

3 + □ = 7   7 = □ + 4

<u>Counting and Combining sets of Objects</u>

Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation).

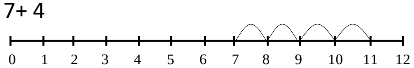
Continue to use C, P, A.



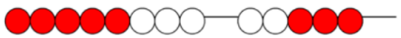
<u>Understanding of counting on with a numbertrack.</u>

<u>Understanding of counting on with a numberline</u> (supported by models and images).

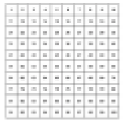
7+ 4



Or using bead string to model how to bridge over 10 by counting on 2 then counting on 3:



'What do you notice?' where partitioning or adjusting is used.



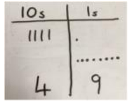
They should use concrete objects such as bead strings and number lines to explore missing number problems e.g 14 + 5 = 10 + □

32 + □ + □ = 100   35 = 1 + □ + 5

It is valuable to use a range of representations (also see Y1).

Use of base 10 to combine two numbers.

Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.



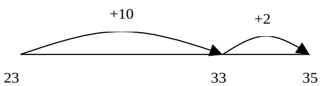
Continue to use number lines to develop understanding of:

Counting on in tens and ones

23 + 12 = 23 + 10 + 2

= 33 + 2

= 35

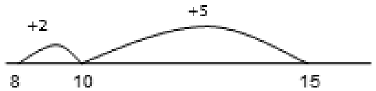


<u>Partitioning and bridging through 10.</u>

The steps in addition often bridge through a multiple of 10

e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

8 + 7 = 15

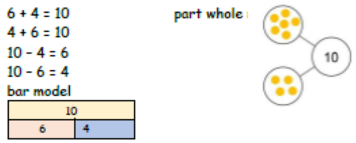


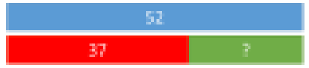
<u>Adding 9 or 11 by adding 10 and adjusting by 1</u>

e.g. Add 9 by adding 10 and adjusting by 1

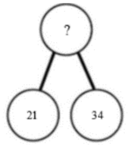
- Add numbers mentally, including: a three-digit number and a single digit number, a 3-digit number and multiples of 10, a 3-digit number and multiples of 100 –
- Estimate the answer to a calculation and use inverse operations to check answers
- Know number pairs that total 1000 (multiples of 100) –
- Calculate 10 or 100 more than any given number

Models continue to be taught to show children how subtraction and addition are related operations.

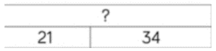




Conceptual variation is continued to be taught.

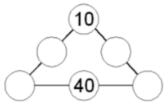


Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.



Children will solve one and two-step addition problems (including missing number problems) using concrete objects and pictorial representations

This number triangle has missing numbers. The numbers along each edge must add up to 90. Put all the numbers: 20, 30, 50 and 60 in the circles to make the totals correct.



Word problems:

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

21 + 34 = 55. Prove it

<u>Partition into tens and ones</u>

Partition both numbers and recombine.

Count on by partitioning the second number only e.g.

247 + 125 = 247 + 100 + 20+ 5

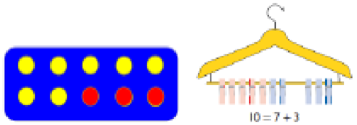
= 347 + 20 + 5

= 367 + 5

= 372

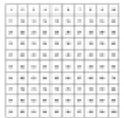
Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

They should see addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 – 3 = 7, understanding of which could be supported by an image like this.



Models can be used to show children how subtraction and addition are related operations.

Number squares are used to show how numbers get larger when others are added to them



(Links between addition and subtraction)

When introduced to the **equals** sign, children should see it as signifying equality.

4 + 1 = 2 + 3

They should become used to seeing it in different positions.

6 + □ = 11

6 + 5 = 5 + □

6 + 5 = □ + 4

Children should become used to seeing equations in different orders and using 'n' to represent the missing number.

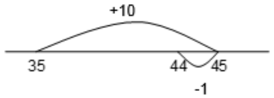
4 = n + 3   4 = 3 + n   3 + n =4   n + 3 = 4

Generalisations

- True or false? Addition makes numbers bigger.
- True or false? You can add numbers in any order and still get the same answer.

(Links between addition and subtraction)

35 + 9 = 44

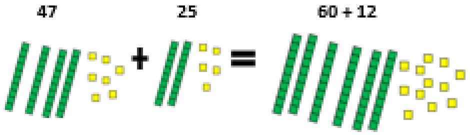


Towards a Written Method

Partitioning in different ways and recombine;

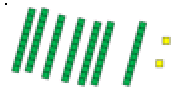
47+25

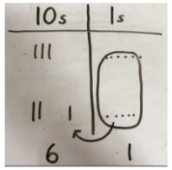
47   **25**   **60 + 12**



Leading to exchanging:

72





Expanded written method

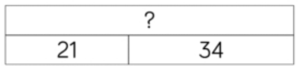
40 + 7 + 20 + 5 =

40+20 + 7 + 5 =

60 + 12 = 72

    40 + 7
  + 20 + 5
    60 + 12 = 72

Bar Models used to solve number problems



Children should learn to check their calculations, by using the inverse. They should continue to see addition as both combining groups and counting on.

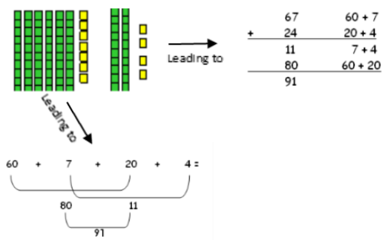
Generalisation

Towards a Written Method

Introduce expanded column addition modelled with Dienes

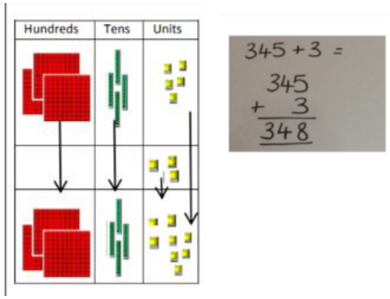
Base 10 equipment:

67 + 24 = 91

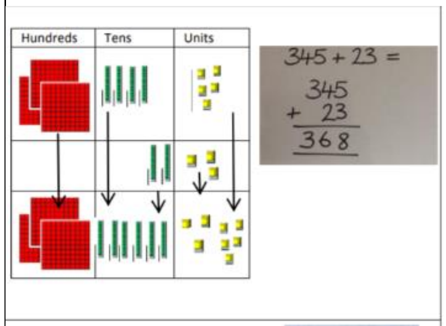


Leading to children understanding the re-grouping between tens and ones. Children use diennes to make the link between practical and those more forma methods.

All children begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.It is important to introduce the method without the need to regroup first and then practise the method where regrouping is required.

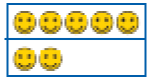


The children set out HTO + O in place value mats and then can transfer this information into base 10 on a PV chart, and then a formal column method. (No Exchanging)

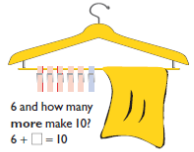


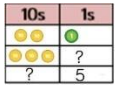
The children set out HTO+TO in place value mats and then transfer this information into the formal column method. (No Exchanging/not crossing tens boundary)

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the [inverse](#) relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.

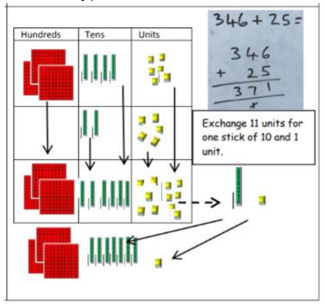


7 + ? = 10

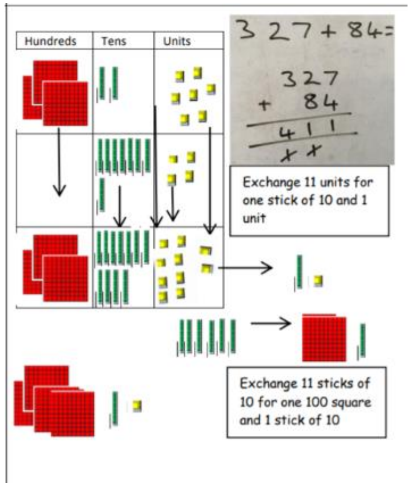




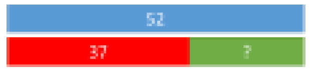
The children set out HTO+TO in place value mats and then transfer this information into the formal column method. (with exchanging/crossing **TENS** boundary)



The children set out HTO+TO in place value mats and then transfer this information into the formal column method. (with exchanging/crossing **HUNDREDS AND TENS** boundary)



<u>Bar Models used to solve number problems</u>



<u>Generalisations</u>

Noticing what happens to the digits when you count in tens and hundreds.
Odd + odd = even etc (see Year 2)
Inverses and related facts – develop fluency in finding related addition and subtraction facts.
Develop the knowledge that the inverse relationship can be used as a checking method.

Addition

Year 4

Missing number/digit problems: C, P, and A models and images continue to be required to support the child's calculation of this.

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should be exposed to using a variety of mental methods: using place value, counting on and using number facts, partitioning etc to:
Add numbers mentally, including: a four digit number and multiples of one thousand –
Use knowledge of doubles to derive related facts (e.g 15 + 16 = 31 because 15 + 15 = 30 and 30 + 1 = 31)
Know number pairs that total 1000 (multiples of 10) - Estimate the answer to a calculation and use inverse operations to check answers

Using place value:
Count in thousands, *e.g. knowing 475 + 200 as 475, 575, 675*
Partitioning, *e.g. 746 + 203 as 700 + 200 and 46 + 3 or 134 + 707 as 130 + 700 and 4 + 7*

Counting on:
Add two 2-digit numbers by adding the multiple of ten then the ones, *e.g. 67 + 55 as 67 add 50 (117) add 5 (122)*
Add near multiples of 10, 100 and 1000, *e.g. 467 + 199 or 3462 + 2999*

Using number facts:
Number bonds to 100 and to next multiple of 100, *e.g. 463 + 37, 1353 + 47*

Number bonds to £1 and to the next whole pound, *e.g. £3.45 + 55p*
Add to next whole number, *e.g. 4.6 + 0.4, 7.2 + 0.8*

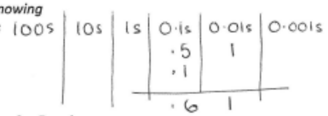
Written methods (progressing to 4-digits)

Year 5

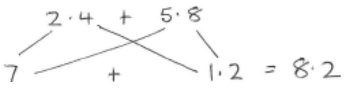
Missing number/digit problems: use the properties of rectangles to deduce related facts and find missing lengths and angles

Mental methods (see Year 4) should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency
e.g. 12462 + 2300 = 14762

Using place value:
Count in 0.1s, 0.01s, *e.g. knowing what 0.1 more than 0.51 is*



Partitioning, *e.g. 2.4 + 5.8 as 2 + 5 and 0.4 + 0.8 and combine the totals: 7 + 1.2 = 8.2*



Counting on:
Add two decimal numbers by adding the ones then the tenths/hundredths, *e.g. 5.72 + 3.05 as 5.72 add 3 (8.72) then add 0.05 (8.77)*
Add near multiples of 1, *e.g. 6.34 + 0.99 or 5.63 + 0.9*
Count on from large numbers, *e.g. 6834 + 3005 as 9834 + 5*

Using number facts:
Number bonds to 1 and to the next whole number, *e.g. 0.4 + 0.6 or 5.7 + 0.3*
Add to next ten from a decimal number, *e.g. 7.8 + 2.2 = 10*

Written methods (progressing to more than 4-digits)
As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

```
  172.83
+  54.68
  227.51
  1  1 1
```

Year 6

Missing number/digit problems: express missing number problems algebraically or with two unknowns.

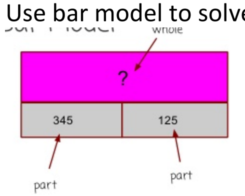
Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods
As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.
Continue calculating with decimals, including those with different numbers of decimal places
Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

Adding several numbers with more than four digits

Problem Solving
Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding.

Use bar model to solve problems:



Add fractions with unlike denominators, *e.g. ¾ + 1/3 = 1 1/12 or 13/12*
2 ¼ + 1 1/3 = 3 7/12

Generalisations
Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BIDMAS, or could be encouraged to design their own ways of remembering.

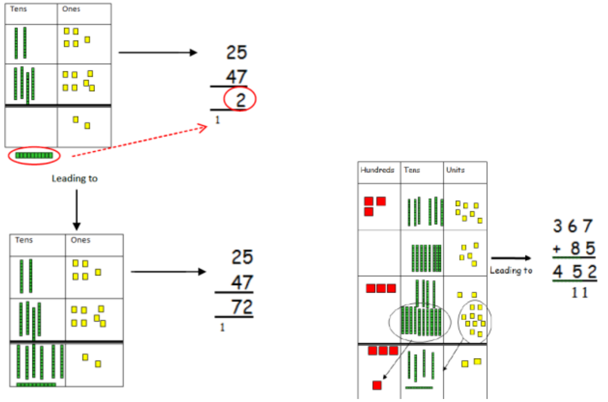
Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.

200 + 40 + 7
100 + 20 + 5
300 + 60 +12 = 372

```
  247
 +125
   12   (7 +5)
   60   (40 + 20)
  300   (200+100)
  372   )
```

Compact written method

Extend to numbers with at least four digits.



Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits). Use and apply this method to money and measurement values too.

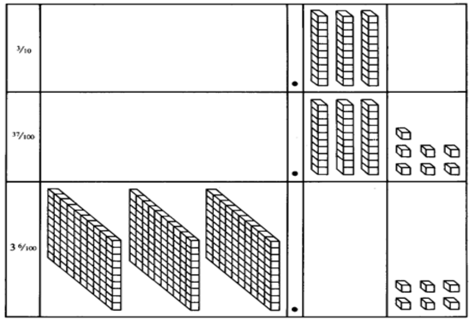
```
  72.8
+ 54.6
 127.4
 1  1
```

Use Bar Models used to solve number problems

Generalisations

Investigate when re-ordering works as a strategy for subtraction. Eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer.

Diennes can be used alongside the columnar method to develop understanding of addition with decimal numbers.



Fraction	100's (Hundreds)	10's (Tens)	1's (Ones)	Decimal Point	1/10 (Tenths)	1/100 (Hundredths)
3/10				•	3	
37/100				•	7	3
8/10				•	8	
80/100				•	8	0
3 6/10			3	•	6	
15 6/100		1	5	•	0	6

Use Bar Models used to solve number problems

Adding fractions with related (like) denominators,

e.g. $\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$

Generalisation

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.

What do you notice about the differences between consecutive square numbers?

Investigate a – b = (a-1) – (b-1) represented visually.

Sometimes, always or never true? Subtracting numbers makes them smaller.

Subtraction

Vocabulary

Subtraction, subtract, take away, distance between, difference between, more than, minus, less than, one less, two less… ten less… one hundred less, one thousand less, equals = same as, most, least, pattern, odd, even, digit, Hundreds, Tens, ones, near multiple of 10, 100 and 1000, tens (hundreds, thousands) boundary, more, one more, two more. ten more. one hundred more, one thousand more, estimate, partition, recombine, difference, decrease, inverse, rounding, column subtraction, exchange, tenths boundary, hundredths boundary, how many more/fewer? Equals sign, is the same as.

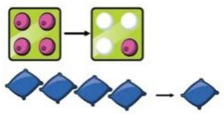
Some Key Questions

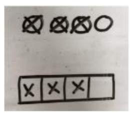
How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many have gone?
One less, two less, ten less… How many fewer is… than…? How much less is…?
What can you see here? Is this true or false? What do you notice? What patterns can you see?
If I know that 7 + 2 = 9, what else do I know? (e.g. 2 + 7 = 9; 9 – 7 = 2; 9 – 2 = 7; 90 – 20 = 70 etc).
When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line, Can you convince me? How do you know?

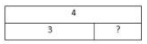
EYFS/Year 1

Use concrete objects and pictorial representations.
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).

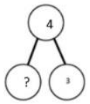
4 – 1 = 3 Children to draw the concrete resources they are using and cross out the correct amount







The bar model can also be used.

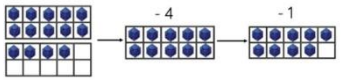


Counting back (using number lines or number tracks) children start with 6 and count back 2.

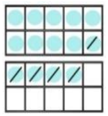
Year 2

Use concrete objects and pictorial representations.
Physically taking away and removing objects from a whole (see EYFS/Y1).

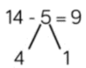
Making 10 using ten frames.
14 – 5



Children to present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.



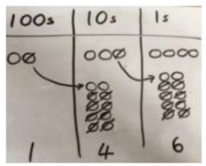
14 – 4 = 10
10 – 1 = 9

Year 3

Use concrete objects and pictorial representations.
Physically taking away and removing objects from a whole (see EYFS/Y1/Y2).

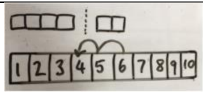
Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving (see EYFS/Y1 and Y2).

Represent the place value counters pictorially; remembering to show what has been exchanged

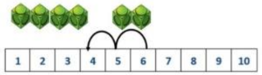


Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

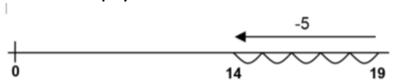
Subtract numbers mentally, including: Subtracting a single digit number from a 3-digit number, Subtracting a multiple of 10 from a 3-digit number, subtracting a multiple of 10 from a 3-digit number

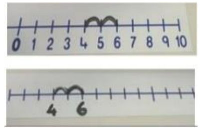


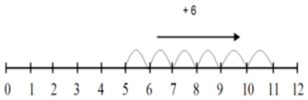
6 – 2 = 4 Children to represent what they see pictorially e.g.



Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line



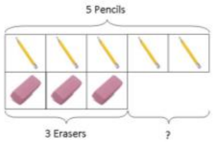




If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown, to empty number lines/number tracks.

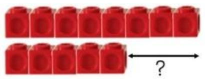
Understand subtraction as take-away:

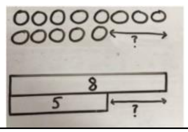
Understand subtraction as finding the difference:



The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation. The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings

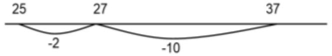
Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate

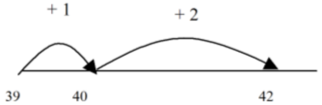




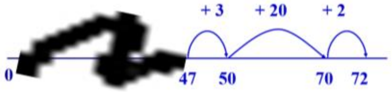
Continue to use number lines to model take-away and difference.

E.g.





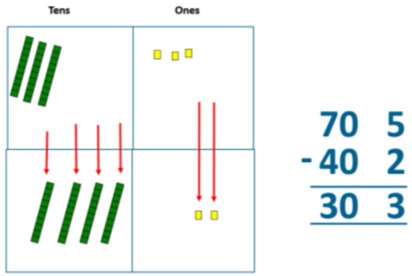
The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



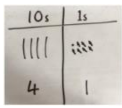
The bar model should continue to be used, as well as images in the context of **measures**.

Towards written methods

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. 75 – 42



Children to represent the base 10 pictorially.



Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting back in tens from any number should lead to subtracting multiples of 10.

Number lines should continue to be an important image to support thinking, for example to model how to subtract 9 by adjusting.

Estimate the answer to a calculation and use inverse operations to check answer

Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

Taking away:

Use place value to subtract, *e.g. 348 – 300 or 348 – 40 or 348 – 8*

Taking away multiples of 10, 100 and £1, *e.g. 476 – 40 = 436, 476 – 300 = 176, £4.76 – £2 = £2.76*

Partitioning, e.g. *68 – 42 as 60 – 40 and 8 – 2 or £6.84 – £2.40 as £6 – £2 and 80p – 40p*

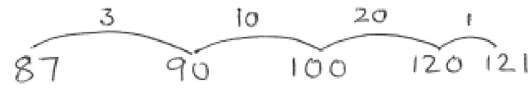
Count back in hundreds, tens then ones, *e.g. 763 – 121 as 763 – 100 (663) then subtract 20 (643) then subtract 1 (642)*

Subtract near multiples, *e.g. 648 – 199 or 86 – 39*

Children should continue to partition numbers in difference ways.

Counting up:

Find a difference between two numbers by counting up from the smaller to the larger, *e.g. 121 – 87*



The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.

Use number bonds:

Number bonds to 100, *e.g. 100 – 35 = 65, 100 – 48 = 52, etc.*

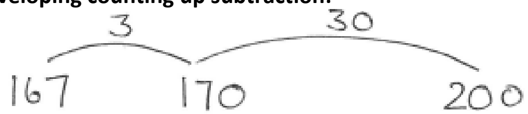
They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g. counting up (difference, or complementary addition) for 201 – 198; counting back (taking away / partition into tens and ones) for 201 – 12.

The strategy of adjusting can be taken further, e.g. subtract 100 and add one back on to subtract 99. Subtract other near multiples of 10 using this strategy.

Missing number problems e.g. □ = 43 – 27; 145 – □ = 138; 274 – 30 = □; 245 – □ = 195; 532 – 200 = □; 364 – 153 = □

Written methods (progressing to 3-digits)

Developing counting up subtraction:



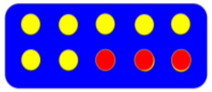
Use counting up to find change:

Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

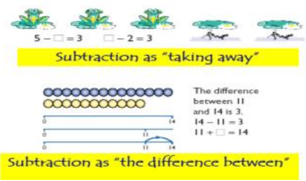
Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions and use of missing numbers.
Missing number problems e.g. 7 = □ - 9; 20 - □ = 9; 15 – 9 = □; □ - □ = 11; 16 – 0 = □

They should see addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 – 3 = 7, understanding of which could be supported by an image like this.



Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones.

Children should begin to understand subtraction as both taking away and finding the difference between, and should find small differences by counting on.

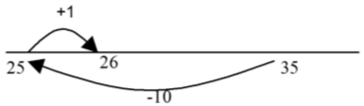


Generalisations

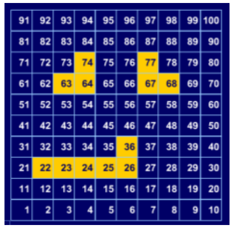
- True or false? Subtraction makes numbers smaller
- When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Children could see the image below and consider, "What can you see here?" e.g.

3 yellow, 1 red, 1 blue. 3 + 1 + 1 = 5

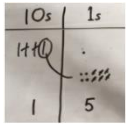


Children should practise subtraction to 20 and 100 to become increasingly fluent. They should use the facts they know to derive others, e.g using 10 - 7 = 3 and 7 = 10 - 3 to calculate 100 - 70 = 30 and 70 = 100 - 30.



As well as number lines, 100 squares could be used to model calculations such as 74 – 11, 77 – 9 or 36 – 14, where partitioning or adjusting are used. On the example above, 1 is in the bottom left corner so that 'up' equates to 'add'.

Represent the base 10 pictorially, remembering to show the exchange.



Children should learn to check their calculations, including by adding to check (the inverse calculation).

They should continue to see subtraction as both take away and finding the difference, and should find a small difference by counting up.

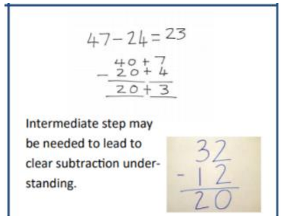
They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. 23 = 20 + 3 = 10 + 13.

Missing number problems, experiencing the = sign in different positions and use of missing numbers e.g. 52 – 8 = □; □ – 20 = 25; 22 = □ – 21; 6 + □ + 3 = 11

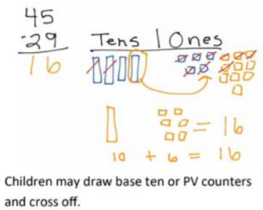
Generalisation

Recognise complements of any fraction to 1, *e.g.* $1 - \frac{1}{4} = \frac{3}{4}$ *or* $1 - \frac{2}{3} = \frac{1}{3}$

Formal column method. Children must understand what has happened when they have crossed out digits.
Introduce expanded column subtraction with no decomposition, modelled with Dienes.
Children begin to set out TO-TO (within the tens boundary/no exchanging).
Subtract ones first and then subtract tens.



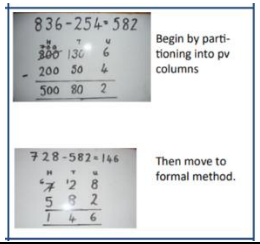
Children begin to set up TO-TO (that cross the tens boundary /exchanging/regrouping (modelled using Dienes).



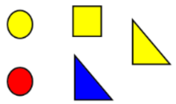
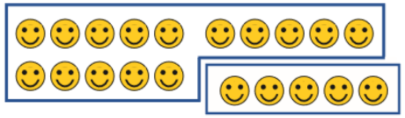
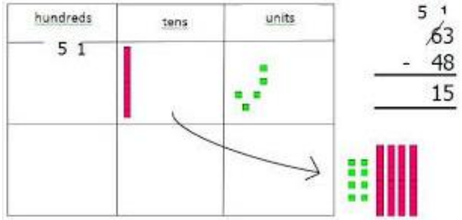
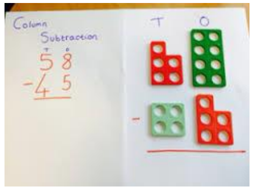
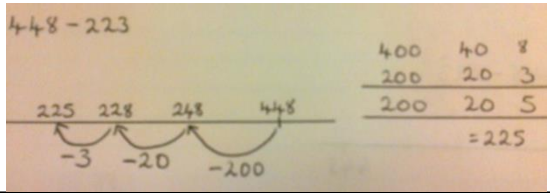
Children set up HTO-TO (within the tens boundary/no exchanging or regrouping) Record as column subtraction.

40	6
- 20	2
22	4

Children begin to set up HTO-TO and HTO-HTO that cross the tens boundary/exchanging/regrouping. Record as column subtraction.



A number line and expanded column method may be compared next to each other.

2 circles, 2 triangles, 1 square. 2 + 2 + 1 = 5  I see 2 shapes with curved lines and 3 with straight lines. 5 = 2 + 3 5 = 3 + 1 + 1 = 2 + 2 + 1 = 2 + 3	• Noticing what happens when you count in tens (the digits in the ones column stay the same) • Odd – odd = even; odd – even = odd; etc • show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot • Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.  15 + 5 = 20	  *Numicon can be used to provide practical support when subtracting.* The formal method should be seen as a more streamlined version of the expanded method, not a new method. **Generalisations** Noticing what happens to the digits when you count in tens and hundreds. Odd – odd = even etc (see Year 2) Inverses and related facts – develop fluency in finding related addition and subtraction facts. Develop the knowledge that the inverse relationship can be used as a checking method. 

Subtraction		
Year 4	**Year 5**	**Year 6**
Missing number/digit problems: 456 + □ = 710; 1□7 + 6□ = 200; 60 + 99 + □ = 340; 200 – 90 – 80 = □; 225 - □ = 150; □ – 25 = 67; 3450 – 1000 = □; □ - 2000 = 900 Missing digit calculations;  	Missing number/digit problems: 6.45 = 6 + 0.4 + □; 119 - □ = 86; 1 000 000 - □ = 999 000; 600 000 + □ + 1000 = 671 000; 12 462 – 2 300 = □ **Mental methods** should continue to develop, supported by a range of models and images, including the number line (see previous year groups.) The bar model should continue to be used to help with problem solving. Children should continue to count regularly, on and back, now including steps of powers of 10.	**Mental Strategies** Consolidate previous years. Children should experiment with order of operations, investigating the effect of positioning the brackets in different places; e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45 **Written methods** Children will use the **compact column subtraction** method with 'exchanging' including problems involving money, measures and decimals with up to two decimal places.

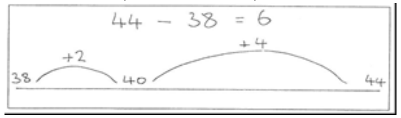
Mental methods should continue to develop, supported by a range of models and images, including the number line (see earlier year groups). The bar model should continue to be used to help with problem solving.

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77



- Reordering: 28 + 75, 75 + 28 (thinking of 28 as 25 + 3)
- Partitioning: counting on or back: 5.6 + 3.7, 5.6 + 3 + 0.7 = 8.6 + 0.7
- Partitioning: bridging through multiples of 10: 6070 – 4987, 4987 + 13 + 1000 + 70
- Partitioning: compensating – 138 + 69, 138 + 70 - 1
- Partitioning: using 'near' doubles - 160 + 170 is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10
- Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm?
- Using known facts and place value to find related facts. Subtract numbers mentally, including:
- Subtracting multiples of one thousand from a 4-digit number
- Use of number pairs that total 1000 (multiples of 10) to calculate subtraction (e.g 1000 – 300 = 700)
- Estimate the answer to a calculation and use inverse operations to check answers

Written methods (progressing to 4-digits)

Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

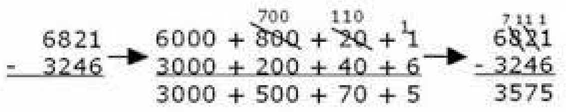
Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55
- Reordering: 4.7 + 5.6 – 0.7, 4.7 – 0.7 + 5.6 = 4 + 5.6
- Partitioning: counting on or back - 540 + 280, 540 + 200 + 80
- Partitioning: bridging through multiples of 10:
- Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 – 0.1
- Partitioning: using 'near' double: 2.5 + 2.6 is double 2.5 and add 0.1 or double 2.6 and subtract 0.1
- Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?
- Using known facts and place value to find related facts.

Written methods (progressing to more than 4-digits)

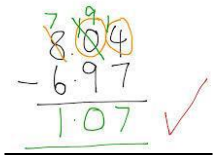
When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.



Children will subtract with decimal values, including mixtures of integers and decimals up to two decimal places, aligning the decimal point.

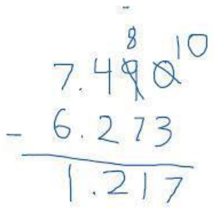
Children who are still not secure with number facts and place value will need to remain on the partitioned column method (or earlier methods) until ready for the compact method.

Ensure children understand a smaller number can be subtracted from a larger number to avoid misconceptions being formed



Generalisation

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.



Teachers may also choose to introduce children to other efficient written layouts which help develop conceptual understanding.

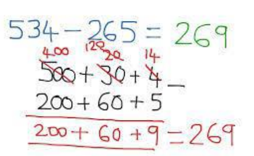
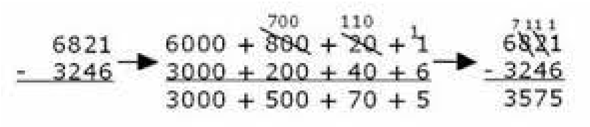
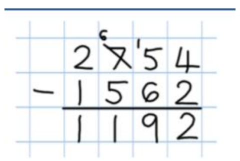
For example:

326
-148
-2
-20
200
178

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BIDMAS, or could be encouraged to design their own ways of remembering.

Sometimes, always or never true? Subtracting numbers makes them smaller.

  Subtract with up to 4 digits. Introduce decimal subtraction through context of money e.g. £27.54- £15.62= £11.92  Use the phrase 'take and make' for exchange **Generalisations** Investigate when re-ordering works as a strategy for subtraction. Eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer.	What do you notice about the differences between consecutive square numbers? [Investigate a – b = (a-1) – (b-1) represented visually].	

Multiplication

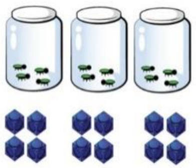
Vocabulary

Ones, groups, lots of, doubling, repeated addition, equal groups of, lots of, times, columns, rows, longer, bigger, higher etc, times as (big, long, wide …etc), times, multiplied by, the product of, multiple, multiply, multiplication array, multiplication tables / facts, grid method altogether, commutative, sets of, once, twice, three times, tens, ones, value, inverse, factor, cube numbers, prime numbers, square numbers, common factors, prime factors, composite numbers

Some Key Questions

Why is an even number an even number? What do you notice? What's the same? What's different?
Can you convince me? How do you know?
How do you know this is a prime number?

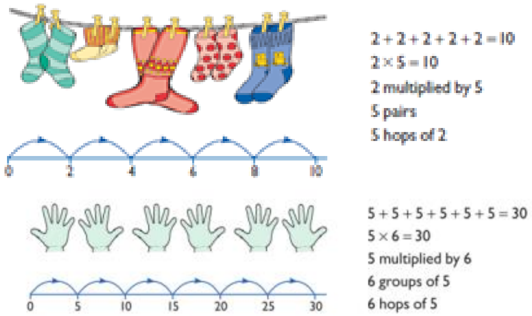
EYFS/Year 1	Year 2	Year 3



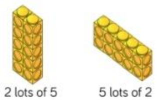
Understand and recognise multiplication is related to making equal groups, doubling and combining groups of the same size. Real life contexts and use of practical equipment. Counting in multiples. Use cubes, Numicon and other objects in the classroom

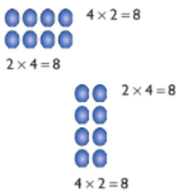
Repeated grouping/repeated addition
4 + 4 + 4
3 × 4
There are 3 equal groups, with 4 in each group.

Washing line, and other practical resources for counting.
Concrete objects. Numicon; bundles of straws, bead strings

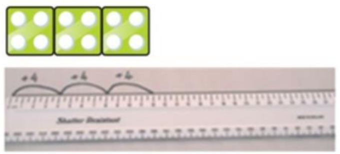


Problem solving with concrete objects (including money and measures. Use Cuisenaire and bar method to develop the vocabulary relating to 'times' –
Pick up five, 4 times
Develop understanding of multiplication using array and to understand multiplication can be done in any order (commutative)



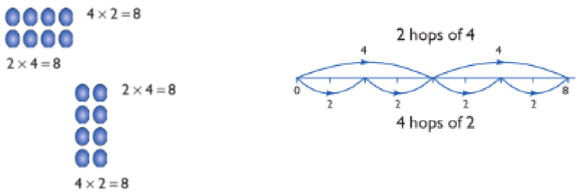


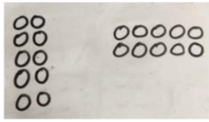
Number lines to show repeated groups-
3 × 4



Continue to count in multiples. Use cubes, Numicon, small objects and other objects in the classroom.

Develop understanding of multiplication using arrays and number lines (see EYFS/Year 1).





Children to represent the arrays pictorially.

Children to be able to use an array to write a range of calculation number sentence using X.
e.g.

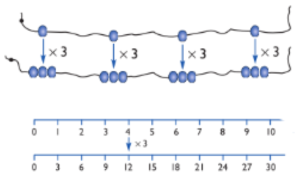
10 = 2 × 5

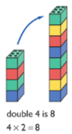
5 × 2 = 10
2 + 2 + 2 + 2 + 2 = 10
10 = 5 + 5

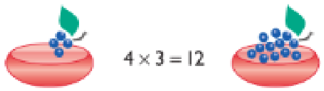
Using understanding of the inverse and practical resources to solve missing number problems.

7 x 2 = ☐	☐ = 2 x 7
7 x ☐ = 14	14 = ☐ x 7
☐ x 2 = 14	14 = 2 x ☐
☐ x ◯ = 14	14 = ☐ x ◯

Include multiplications not in the 2, 5 or 10 times tables.
Begin to develop understanding of multiplication as scaling (3 times bigger/taller)



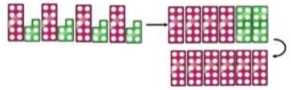




Continue to count in multiples. Use concrete manipulatives and pictorial methods to develop understanding of multiplication using arrays, number lines and a range of equations (see EYFS/Year 1/2).

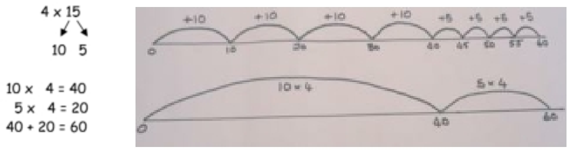
Mental methods
Doubling 2 digit numbers using partitioning
Demonstrating multiplication on a number line – jumping in larger groups of amounts;
13 x 4 = 10 groups 4 and 3 groups of 4

Partition to multiply using Numicon, base 10 or Cuisenaire rods.
4 × 15

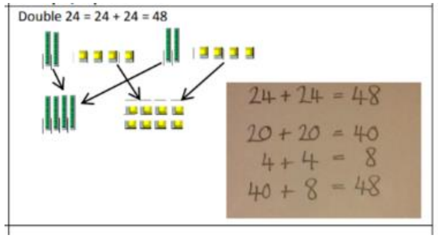


Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings and drawings to solve problems should be encouraged.

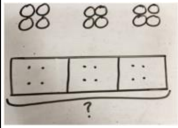


Children should practise times table facts
3 x 1 =
3 x 2 =
3 x 3 =
Count forwards and backwards in multiples of 4, 8, 50 & 100
Know the 3, 4 and 8 times tables (in and out of order)
Connect the 2, 4 and 8 times tables through doubling
Use knowledge of place value to calculate multiplication (e.g. 2 x 2 = 4, 2 x 20 = 40, 2 x 200 = 400)
Children will learn to calculate doubles of 2-digit numbers through partitioning



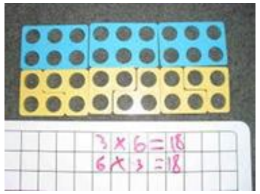
Missing number problems with the equal sign in different places.
Written methods (progressing to 2d x 1d)

Children to represent the practical resources in a picture and use a bar model.



Children should begin to understand multiplication as scaling in terms of double and half. (e.g. that towers of cubes is double the height of the others)

Use numicon to teach facts:

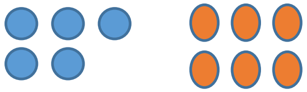


Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

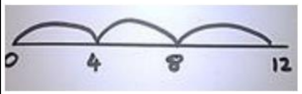
Children should memorise and reason with numbers in 2, 5 and 10 times tables

They should see ways to represent odd and even numbers. This will help them to understand the pattern in numbers.



Abstract number line showing three jumps of four.

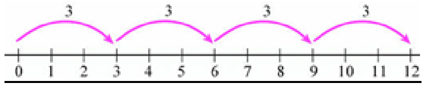
3 × 4 = 12



Doubling numbers up to 10 + 10 Link with understanding scaling
Using known doubles to work out double 2d numbers by partitioning into the tens and ones.
(double 15 = double 10 + double 5)

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10.
Number lines should continue to be an important image to support thinking, for example

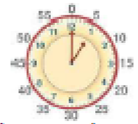


3 x 4 = 12

Children should practise times table facts

2 x 1 =
2 x 2 =
2 x 3 =

Use a clock face to support understanding of counting in 5s.



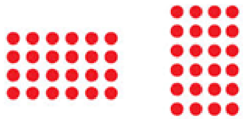
Use money to support counting in 2s, 5s, 10s, 20s, 50s

Use of CLIC Smile multiplication-see.

Smile Multiplication

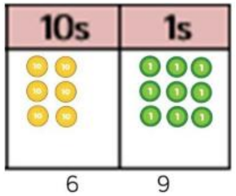
L.I: I can multiply multiples of 10 using the smile method.
x2
x2, x5, x10
x2, x5, x10, x3, x4

Counters in arrays used to teach commutativity



4 x 6 = 24   6 x 4 = 24

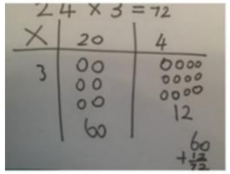
Children can represent their work with place value counters in a way that they understand.



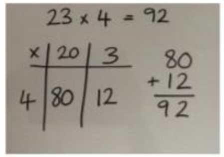
They can draw the counters using colours to show different amounts.

With place value counters (base 10 can also be used.) 3 × 23

Or use the circles in the different columns to show their thinking as shown below.

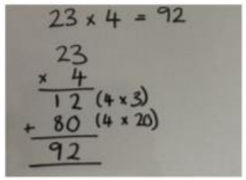


Children will be taught to multiply numbers (TO x O) through partitioning and the formal written method of grid multiplication

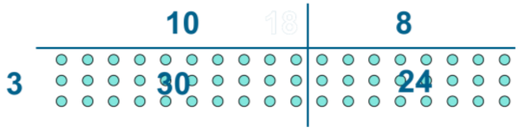


Children will be taught to multiply numbers (TO x O) using the formal written method of expanded column multiplication and make the link to grid method

Children to multiply TO x O and TO x TO using CLIC Smile multiplication (see Year 2).



Developing written methods using understanding of visual images



Develop onto the grid method

	10	8
3	30	24

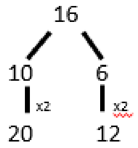
Generalisations

Understand 6 counters can be arranged as 3+3 or 2+2+2

Understand that when counting in twos, the numbers are always even.

Towards written methods

Use jottings to develop an understanding of doubling two digit numbers. Partitioning into the tens and ones.



Generalisation

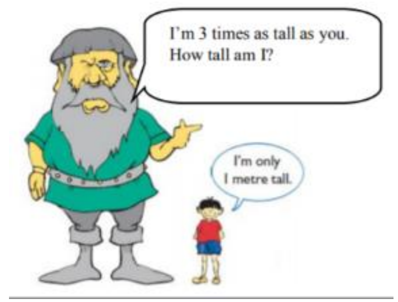
Commutative law shown on array

Repeated addition can be shown mentally on a number line

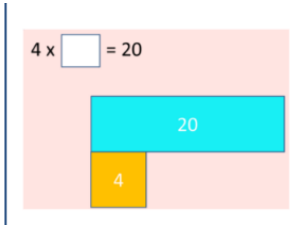
Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups.

Give children opportunities for children to explore this and deepen understanding using Dienes apparatus.

Children will solve problems involving multiplication, including scaling



Bar model are used to explore missing numbers



Generalisations

Connecting x2, x4 and x8 through multiplication facts

Comparing times tables with the same times tables which is ten times bigger. If 4 x 3 = 12, then we know 4 x 30 = 120. Use place value counters to demonstrate this.

When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

Multiplication

Year 4

Continue with a range of concrete, pictorial and abstract strategies as in earlier years but with appropriate equations and numbers. Also include equations with missing digits

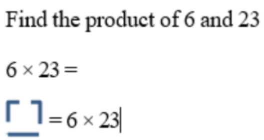
☐2 x 5 = 160

Mental methods

Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

Year 5

Continue with a range of concrete, pictorial and abstract strategies as in earlier years but with appropriate equations and numbers. Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits.




```
  6      23
× 23    × 6
____    ___
```

Year 6

Continue with a range of strategies and equations as in previous years but with appropriate numbers. Also include equations with missing digits

Mental methods

Identifying common factors and multiples of given numbers

Solving practical problems where children need to scale up. Relate to known number facts.

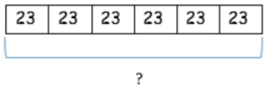
Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5;
(20 – 5) x 3 = 45

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.
Become fluent and confident to recall all tables to x 12
Use the context of a week and a calendar to support the 7 times table (e.g. how many days in 5 weeks?)
Use of finger strategy for 9 times table.

Multiply 3 numbers together e.g.
When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.

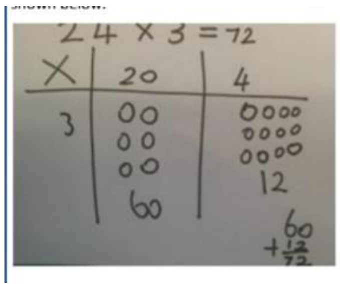


They should be encouraged to choose from a range of strategies:

- Partitioning using x10, x20 etc
- Doubling to solve x2, x4, x8
- Recall of times tables
- Use of commutativity of multiplication

Know all times tables up to and including 12 x 12 (by the end of Year 4)
Recognise and use factor pairs (e.g factor pairs for numbers up to and including 10) –
Know that TO x 5 is TO x 10 then divide by 2 (e.g 18 x 5 = (18 x 10) ÷ 2 = 90)
Know that TO x 9 is TU x 10 then subtract TO (e.g 18 x 9 = (18 x 10) – 18 = 162)

Grid method recap for 2 digits x 1 digit



Written methods (progressing to 3d x 2d)

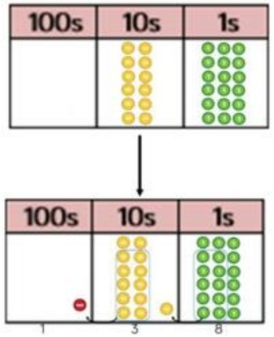
Mental methods

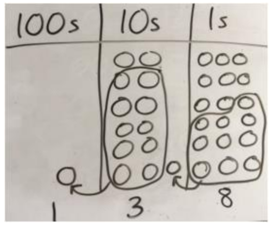
Children should continue to count regularly, on and back, now including steps of powers of 10.
Multiply by 10, 100, 1000, including decimals (Moving Digits ITP)

Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35)

Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning)
Solving practical problems where children need to scale up. Relate to known number facts.
Identify factor pairs for numbers

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.





They should be encouraged to choose from a range of strategies to solve problems mentally:

- Partitioning using x10, x20 etc
- Doubling to solve x2, x4, x8
- Recall of times tables
- Use of commutativity of multiplication

If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)

Written methods (progressing to 4d x 2d)
Long multiplication using place value counters

They should be encouraged to choose from a range of strategies to solve problems mentally:

- Partitioning using x10, x20 etc
- Doubling to solve x2, x4, x8
- Recall of times tables
- Use of commutativity of multiplication

If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)

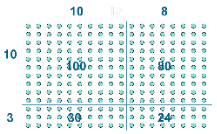
Written methods
Continue to refine and deepen understanding of written methods including fluency for using long multiplication

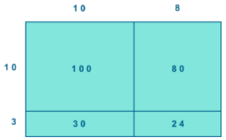
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

```
  2 3  1
   1 3 4 2
x      1 8
 1 3 4 2 0
 1 0 7 3 6
 2 4 1 5 6
   1
```

Generalisations
Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BIDMAS, or could be encouraged to design their own ways of remembering.
Understanding the use of multiplication to support conversions between units of measurement.

Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters.





123 x 5

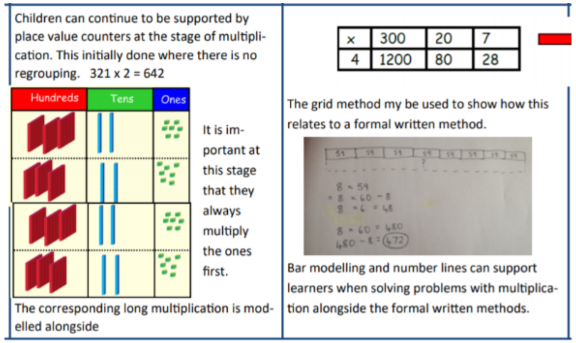
x	100	20	3
5	500	100	15

500
\+ 100
\+ 15
615

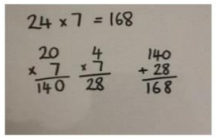
Encourage column addition to add accurately.

Column multiplication

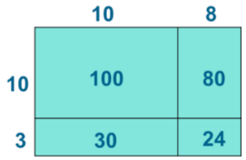
Pupils will move onto the short multiplication method if and when children are confident and accurate multiplying two and three-digit numbers by a one-digit this way, and are already confident in 'carrying' for written addition.



Children will be taught to multiply numbers (TO x O) by partitioning the 2- digit number and using two short multiplications along with addition to solve the problem (Distributive Law)



Children to explore how the grid method supports an understanding of long multiplication (for 2d x 2d)



		1	8			
	×	1	3			
	1	8	0			
		5	4			
	2	3	4			

Long multiplication method is used to multiply numbers with at least four-digits by a two-digit number**.** It can be introduced alongside the grid method as before.

X	2000	400	50	1	
60	120000	24000	3000	60	147,060
3	6000	1200	150	3	7353

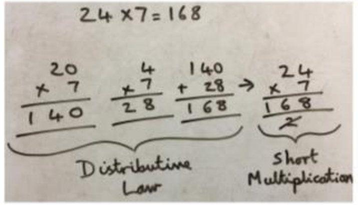
2451
x 63
7353
147060
1 2 3

2451 X 3 on the first row, numbers are carried
2451 X 60 on the next row.

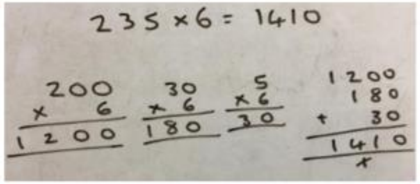
Generalisation

Relating arrays to an understanding of square numbers and making cubes to show cube numbers.
Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000)

Children will be taught to multiply numbers (TO x O) using the formal written method of short multiplication and will link with the Distributive Law method

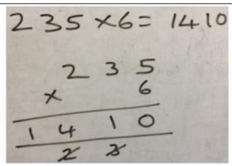


Children will be taught to multiply numbers (HTO x O) by



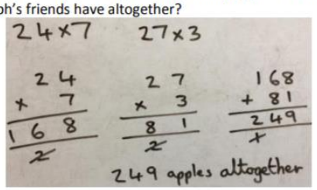
partitioning the 3- digit number and using two short multiplications along with addition to solve the problem

Children will be taught to multiply numbers (HTO x O) using the formal written method of short multiplication and will link with the Distributive Law method



Solve problems involving multiplying and adding to multiply two or threedigit numbers by one digit

Harriet has 7 friends who each have 24 apples. Joseph has 3 friends who each have 27 apples. How many apples do Harriet and Joseph's friends have altogether?



Continue to build on Smile Multiplication to multiply HTO x TO and HTO x HTO

Pupils need to begin to approximate before they calculate, and make this a regular part of their calculating

by going back to the approximation to check the reasonableness of their answer. E.g. 346 x 9 is approximately 350 x 10 = 3500. **<u>Generalisations</u>** Children given the opportunity to investigate numbers multiplied by 1 and 0. When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)		

Division

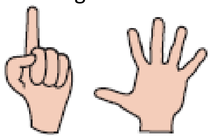
Vocabulary

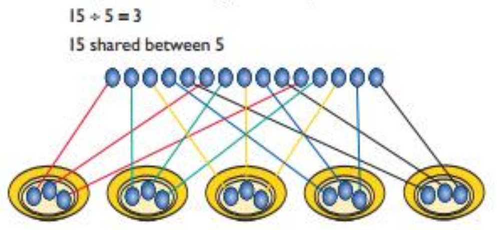
group in pairs, 2s, 5s, 3s … 10s etc, equal groups of, share, share between, group, divide, ÷, divided by, divided into, divisible by, remainder, half, quarter, fraction, one each, two each, left, left over, halves, third, lots of, array, equivalent, inverse, groups of, factor, factor pair, common factors, multiple, times as (big, long, wide …etc), equals, remainder, quotient, divisor, prime number, prime factors composite numbers, short division, square number, cube number, power of

Some Key Questions

How many groups of…? How many in each group? Share… equally into…What can do you notice? How many 10s can you subtract from 60? I think of a number and double it. My answer is 8. What was my number? If 12 x 2 = 24, what is 24 ÷ 2? Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?), Questions that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?)

What is the missing number? 17 = 5 x 3 + __ , __ = 2 x 8 + 1

EYFS/Year 1	Year 2	Year 3
Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10. Children should be given opportunities to reason about what they notice in number patterns. Children need to see and hear representations of division as both grouping and sharing. Division can be introduced through halving and linked to real life contexts. **<u>Group</u>** AND **<u>share small quantities- understanding the difference between the two concepts.</u>** **<u>Sharing</u>** Develops importance of one-to-one correspondence.	Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Children need to see and hear representations of division as both grouping and sharing. Division can be introduced through half, quarter, third and linked to real life contexts. Children who are able to count in twos, threes, fives and tens can use this knowledge to work out other facts such as 2 × 6, 5 × 4, 10 × 9. Show the children how to hold out their fingers and count, touching each finger in turn. So for 2 × 6 (six twos), hold up 6 fingers:  Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.	Children should count regularly, on and back, in steps of 3, 4 and 8. Children are encouraged to use what they know about known times table facts to work out other times tables. Continue to make representations linked with real life contexts. This then helps them to continue to make new connections (e.g. through doubling they make connections between the 2, 4 and 8 times tables). Children will make use of multiplication and division facts they know to make links with other facts. 3 x 2 = 6, 6 ÷ 3 = 2, 2 = 6 ÷ 3 30 x 2 = 60, 60 ÷ 3 = 20, 2 = 60 ÷ 30 They should be given opportunities to solve grouping and sharing problems practically (including where there is a remainder but the answer needs to given as a whole number) e.g. Pencils are sold in packs of 10. How many packs will I need to buy for 24 children?

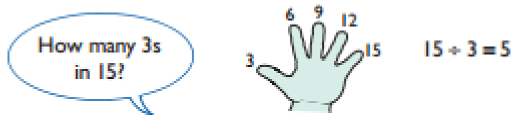


Children should be taught to share using concrete apparatus, then pictorially.

Sharing – 6 sweets are shared between 2 people. How many do they have each?

Grouping

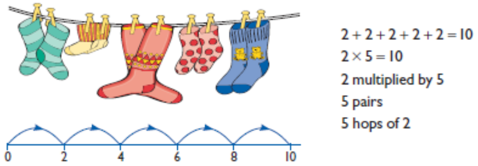
Children should apply their counting skills to develop some understanding of grouping.



Grouping-

How many 2's are in 6?

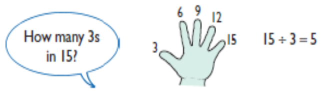
They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.



They should use objects to group and share amounts to develop understanding of division in a practical sense. E.g. using Numicon to find out how many 5's are in 30? How many pairs of gloves if you have 12 gloves? Share 6 objects into 2 groups;

This can then be used to support finding out 'How many 3's are in 18?' and children count along fingers in 3's therefore making link between multiplication and division.

Children should continue to develop understanding of division as sharing **and** grouping using practical apparatus, arrays and pictorial representations. .

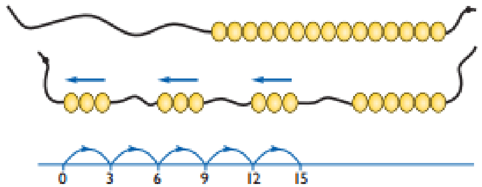


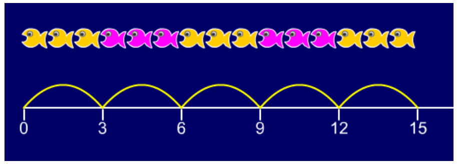
15 pencils shared between 3 pots, how many in each pot?

Grouping using a numberline

Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'.

15 ÷ 3 = 5

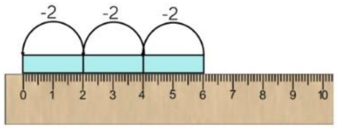




Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?

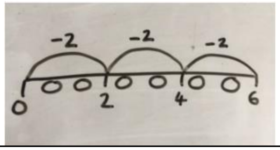
Repeated subtraction using Cuisenaire rods above a ruler.

6 ÷ 2



3 groups of 2

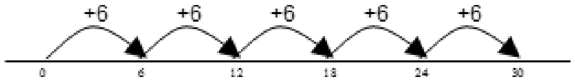
Children to represent repeated subtraction pictorially.



Grouping

How many 6's are in 30?

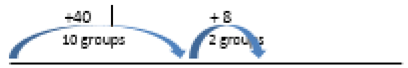
30 ÷ 6 can be modelled as:

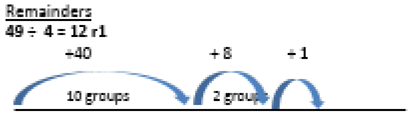


Becoming more efficient using a numberline

Children need to be able to partition the dividend in different ways.

48 ÷ **4** = **12**





Sharing – 49 shared between 4. How many left over?

Grouping – How many 4s make 49. How many are left over?

Place value counters can be used to support children apply their knowledge of grouping.

For example:

60 ÷ 10 = How many groups of 10 in 60?

600 ÷ 100 = How many groups of 100 in 600?

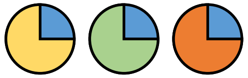
÷ = signs and missing numbers

Continue using a range of equations as in year 2 but with appropriate numbers.

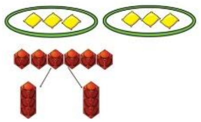
Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure.

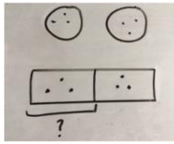
Use children's intuition to support understanding of fractions as an answer to a sharing problem.

3 apples shared between 4 people = $\frac{3}{4}$

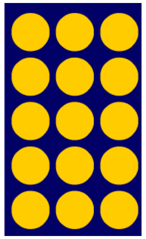


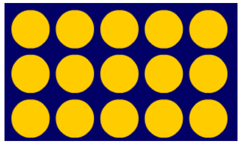
Children should make connections between multiplication and division using arrays





Use of arrays as a pictorial representation for division.
15 ÷ 3 = 5 There are 5 groups of 3.
15 ÷ 5 = 3 There are 3 groups of 5.





Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.

E.g.*16 children went to the park at the weekend. Half that number went swimming. How many children went swimming?*

Generalisations

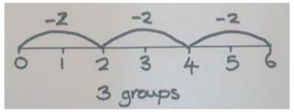
- True or false? I can only halve even numbers.
- Grouping and sharing are different types of problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing.

6 ÷ 2 = 3

3	3

Children should also be encouraged to use their 2 times tables facts.

Abstract number line to represent the equal groups that have been subtracted.



÷ = signs and missing numbers

6 ÷ 2 = □	□ = 6 ÷ 2
6 ÷ □ = 3	3 = 6 ÷ □
□ ÷ 2 = 3	3 = □ ÷ 2
□ ÷ ∇ = 3	3 = □ ÷ ∇

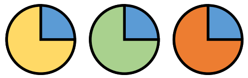
Know and understand sharing and grouping- introducing children to the ÷ sign.

Children should be given opportunities to find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing.

They will explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half.

Use children's intuition to support understanding of fractions as an answer to a sharing problem.

3 apples shared between 4 people = $\frac{3}{4}$

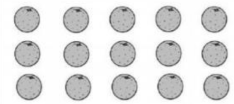


Generalisations

Noticing how counting in multiples if 2, 5 and 10 relates to the number of groups you have counted (introducing times tables)

An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?)

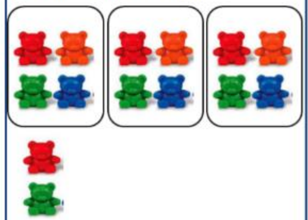
Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.

 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Children use their multiplication facts to solve division with remainders

14 ÷ 3 =

Divide objects between groups and see how much is left over



2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.
13 ÷ 4

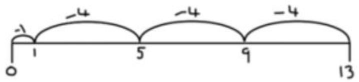
Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.

There are 3 whole squares, with 1 left over.

13 ÷ 4 – 3 remainder 1

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

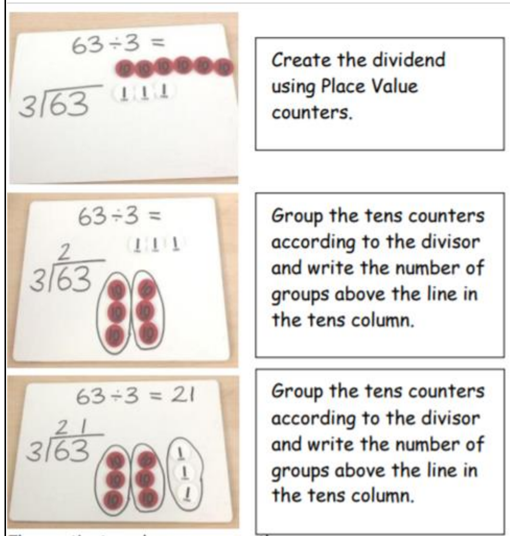


Complete written divisions and show the remainder using r.

29 ÷ 8 = 3 REMAINDER 5
↑ dividend ↑ divisor ↑ quotient ↑ remainder

Written method

Children will use practical resources to support the short division method and will be encouraged to use multiples of the divisor to assist (TO ÷ O)



The quotient can be seen across the groups.

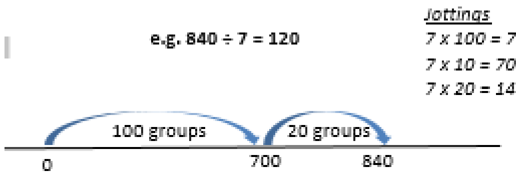
Generalisations
Inverses and related facts – develop fluency in finding related multiplication and division facts.
Develop the knowledge that the inverse relationship can be used as a checking method.

Division

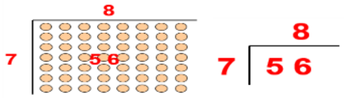
Year 4	Year 5	Year 6
Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 25 and 1000. Children should learn the multiplication facts to 12 x 12. **÷ = signs and missing numbers** Continue using a range of equations as in year 3 but with appropriate numbers. **Sharing, Grouping and using a number line** Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations: • Using tables facts with which they are fluent • Experiencing a logical progression in the numbers they use, for example: 1. Dividend just over 10x the divisor, e.g. 84 ÷ 7	Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency. Children should practice and apply the multiplication facts to 12 x 12. **Formal Written Methods** Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used (see link from fig. 1 in Year 4) E.g. 1435 ÷ 6	Children should count regularly, building on previous work in previous years. Children should practice and apply the multiplication facts to 12 x 12. **÷ = signs and missing numbers** Continue using a range of equations but with appropriate numbers **Grouping and using a number line** Children will continue to explore division as grouping, and to represent calculations on a number line as appropriate. Quotients should be expressed as decimals and fractions **Formal Written Methods – long and short division** E.g. 1504 ÷ 8

2. Dividend just over 10x the divisor when the divisor is a teen number, e.g. 173 ÷ 15 (learning sensible strategies for calculations such as 102 ÷ 17)
3. Dividend over 100x the divisor, e.g. 840 ÷ 7
4. Dividend over 20x the divisor, e.g. 168 ÷ 7

All of the above stages should include calculations with remainders as well as without.
Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)
Figure 1:

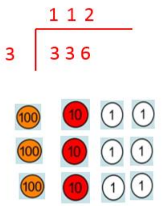


Towards a formal written method
Alongside pictorial representations and the use of models and images, children should progress onto short division using a bus stop method.

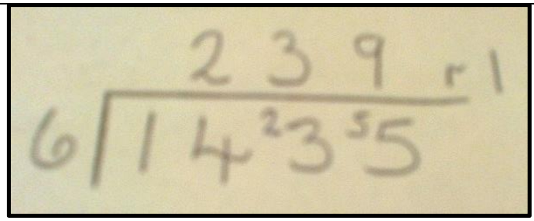


Dienes can be used to support children apply their knowledge of grouping. Reference should be made to the value of each digit in the dividend.

Each digit as a multiple of the divisor
'How many groups of 3 are there in the hundreds column?'
'How many groups of 3 are there in the tens column?'
'How many groups of 3 are there in the units/ones column?'



Formal Written Methods
Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)

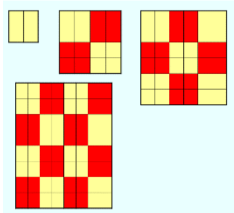


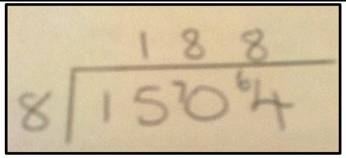
Children begin to practically develop their understanding of how to express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

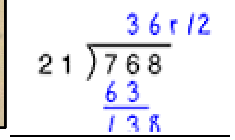
Generalisations
The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g.
Start: **24 = 24**
Player 1: **4 x 6 = 24**
Player 2: **4 x 6 = 12 x 2**
Player 1: **48 ÷ 2 = 12 x 2**

Sometimes, always, never true questions about multiples and divisibility. E.g.:

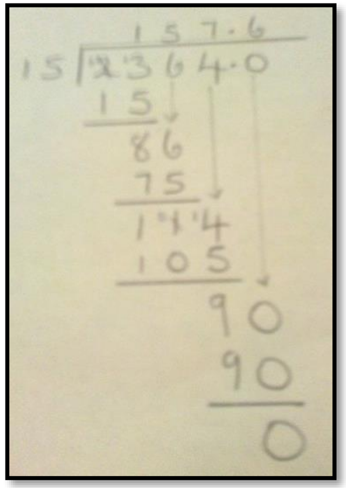
- If the last two digits of a number are divisible by 4, the number will be divisible by 4.
- If the digital root of a number is 9, the number will be divisible by 9.
- When you square an even number the result will be divisible by 4 (one example of 'proof' shown left)

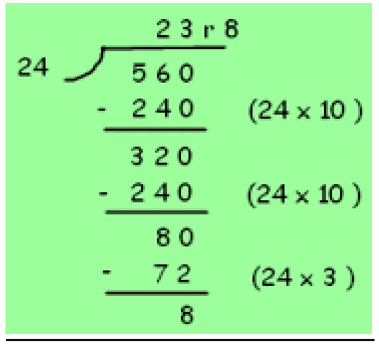




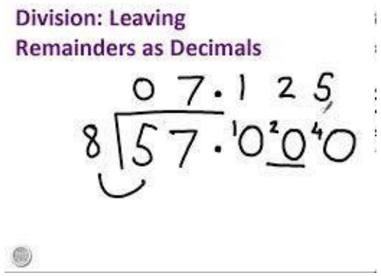


E.g. 2364 ÷ 15

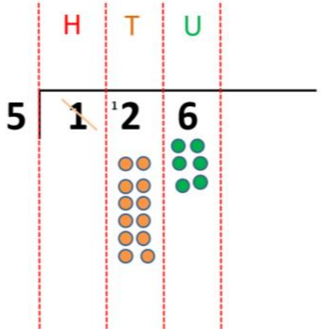


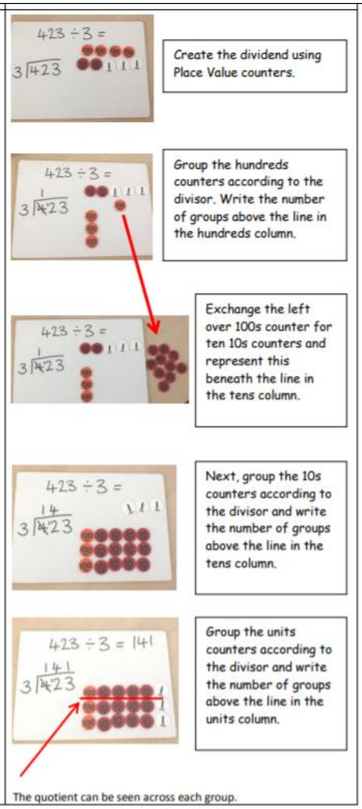


Pupils continue to develop short division, including answers as a decimal. They continue to use this method, but with numbers to at least four-digits and understand how to express remainders as fractions, decimals, whole number remainders or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.



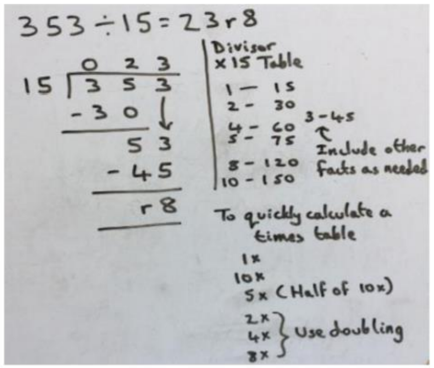
Generalisations
Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BIDMAS, or could be encouraged to design their own ways of remembering.

Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1  When children have conceptual understanding and fluency using the bus stop method without remainders, they can then progress onto ‘carrying’ their remainder across to the next digit. Children will use practical resources to support solving division number sentences with remainders (HTO ÷ O)  **<u>Children will use practical resources to support the short division method where exchange across place value columns occurs. (HTO ÷ O)</u>**		Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5) Using what you know about rules of divisibility, do you think 7919 is a prime number? Explain your answer.

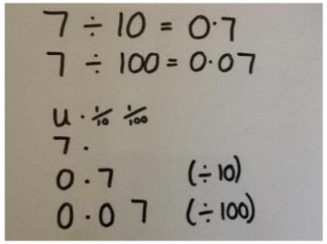


The quotient can be seen across each group.

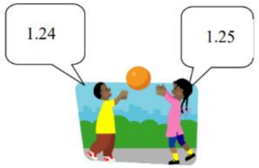
Children will use the short division method where exchange across the place value columns occurs. Pupils will be encouraged to use multiples of the divisor to assist (HTO ÷ TO)



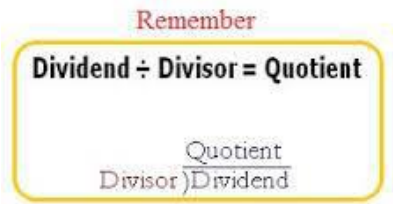
Find the effect of dividing a 1 or 2-digit number by 10 and 100; identifying the value of the digits in the answer as units, tenths and hundredths



Count up and down in hundredths; recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten



What should I cut my pizza into if I have 100 people to serve?



<u>Generalisations</u>

True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?

Is it sometimes, always or never true that □ ÷ Δ = Δ ÷ □?

Inverses and deriving facts. 'Know one, get lots free!' e.g.: 2 x 3 = 6, so 3 x 2 = 6, 6 ÷ 2 = 3, 60 ÷ 20 = 3, 600 ÷ 3 = 200 etc.

Sometimes, always, never true questions about multiples and divisibility. <u>(When looking at the examples on this page, remember that they **may not** be 'always true'!)</u> E.g.: • Multiples of 5 end in 0 or 5. • The digital root of a multiple of 3 will be 3, 6 or 9. • The sum of 4 even numbers is divisible by 4.		